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Weight control with using goal programming in data envelopment analysis

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ABSTRACT

Due to the volume of information, external effects on performance, intense global competition, limited number of units in relation to good decisions, sudden changes of policy passive approach, with acute problems, among the factors which of the impact their income is not suitable solution to improve efficiency. Nowadays Data Envelopment Analysis is one of the sciences which could consider, And getting great strides with its progress in terms of caring out of improved performance. One of the disadvantages of this science is estimating of efficiency at best that accrue by weighting the input and output. (This means that a single decision-maker assigns the strengths weighs of top and weaknesses weights of lower). However, extensive research has been done in data envelopment analysis. But the ways in which resource allocation problem, compared with other issues such as weight control or target selection is very small. In this paper, we investigate the relationship between resource allocation problems with the problem of weight control and target selection.

1. Introduction

For the first time Farrell in 1957, present a model for the assessment and calculation of efficiency with multiple inputs and a single output. Almost two decades after Charnes, cooper, Rhodes, in 1978, this technique is generalized to multiple output data and they named it envelopment analysis.

Data envelopment analysis is an efficient measurement method, in which a set of decision units consist of multiple input and output are there.

In many parts of the world for evaluating the performance of institutions and other common activities in different fields, different application of data envelopment analysis (DEA) has been accessed.

Publishing thousands of articles and books on data envelopment analysis DEA is a kind of proof for this claim.

DEA is also possible to operate the new approach to other methods of evaluation have been previously provided.

Some advantages of DEA are:

- It is able to specify the separate sources and A) amounts of inefficiency in each input and each output of each entity (hospitals, airports, etc.).
- It is able to model a set of work B) benchmarking that has been used for specifying the evaluation and determination of the sources of inefficiency.

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In organizational management using data envelopment analysis and its integration with economic theories and the terms of existing facilities is highly regarded e.g., the allocation of resources or control weight.

The first model (DEA) under the CCR in 1978 is presented. The model name was taken from the names of the top providers of Charnes, cooper, Rhodes and with specific approach of the effectiveness of the proposed is assessed and necessary to enhance the efficiency of inefficient units and deliver them to the efficient frontier of offers.

Max $Z_0 = \sum_{r=1}^{s} u_r y_{ro}$ S.t. $\sum_{i=1}^{m} v_i X_{io} = 1$ $\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0$ $u_r, v_i \ge 0$, j=1,2,...,n X_{ii}: the amount of input i to unit j (i = 1,2, ..., m) y_{ri} : r output for unit j (r = 1,2, ..., s) u_r: r output weight v_i: the weight of input i

Unreasonable weight problem occurs when it assign the large model weight to an output or very small weight to an input whereas this is unreasonable and inappropriate.

For the first time Roll (1991), examined the issue of common weights. In summary, the aim of this research was to develop a model that only through a weight for each input and output parameters

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obtained and to calculate and compare the performance on the basis of joint action.

2. Programming with multiple targets

Goal programming approach presented in 1955 by Charnes, cooper. The model for each of the objective functions is a set of ideal values and priorities of the objectives sought to minimize the deviation with respect to numbers of ideal goals of the goal programming.

Based on the strong foundations of mathematical knowledge in operations research, decision-making is widely used in various fields. These two branches of knowledge applied to the problem of measuring the relative efficiency DEA is a set of matching units with multiple inputs and outputs, which, according to the creators of many research centers are working on it. Another branch of operations research models for decision-making with multiple targets that will help a lot in deciding which of several different and conflicting aims are met.

In general, the purpose of programming can be achieved by a certain amount, as program

In general, the purpose of programming can be achieved by a certain amount, as program goals that may even achieving these values with different priorities must be considered.. In such cases, the planning can be used as a target or multiple targets. However, this method is similar, except that linear programming can be conflicting goals together.

This can encompass multiple objectives based on minimizing the deviation from the target set.

Kornbluth (1991) first stated that data envelopment analysis model could be consider could be consider as a linear fractional multi objective problem.

General from of this model is as follows:

$Min \left[\sum_{i=1}^{k} (d_{j}^{+} + d_{j}^{-})\right]$) ^p] ^{1/p}	
S.t. $g_i(x) \leq 0$,	i=1,,m
$f_{j}(x) + d_{j} - d_{j} + b_{j}$,	j=1,,k
d_{j^-} , $d_{j^+} \ge 0$,	j=1,,k
$d_j \sim d_j = 0$,	j=1,,k

Where, f_j represents the objectives, b_j is goal values of objectives and dj^+ , dj^- are deviations above and lower that jth goal, respectively. P values indicate the priorities of goals in respect to each other defining by decision maker.

3. Model for calculating the common weights

For beginning we consider the problem MOFP:

$$\begin{aligned} & \text{Max} \quad W = \left\{ \frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \frac{\sum_{r=1}^{s} u_r y_{r2}}{\sum_{i=1}^{m} v_i x_{i2}}, \dots, \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \right\} \\ & \text{S.t.} \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \quad , \quad \forall j \qquad (1) \\ & u_r \, , \, v_i \geq \varepsilon \quad , \quad \forall i, j \end{aligned}$$

Goal programming (GP), a method is proposed to solve the above problem (Tamiz et al., 1998).

Using Gp model (1) can be non-linear model to identify a common set of weights, we convert (e.g., Davoodi et al., 2012).

Min
$$\sum_{j=1}^{n} (\varphi_j^- + \varphi_j^+)$$
 (2)

S.t.
$$\frac{\sum_{r=1}^{n} u_r y_{rj}}{\sum_{r=1}^{m} v_r x_{ij}} + \varphi_j^- + \varphi_j^+ = A_j$$
 (2a)

$$\begin{split} & \sum_{r=1}^{s} u_r y_{rj} \\ & \sum_{i=1}^{m} v_i x_{ij} \\ & \varphi_j^-, \varphi_j^+ \ge 0 \quad , \quad \forall i \ , u_r \ge \varepsilon \quad , \quad \forall i, j, r \end{split}$$

 ϕ_j^- And ϕ_j^+ , respectively, negative and positive deviations from target j is called deviation. In addition A_j , the model (2) is number one because it wants to rank the efficiency of each DMUs is the same.

Given the constraints (2b) and positive deviation variables φ_j^+ Do not be constraint (2a) takes a positive value. $\varphi_j^+ = 0$ There should be restrictions on the form (2a) can be rewritten as follows:

$$\sum_{r=1}^{s} u_{r} y_{rj} + \varphi_{j}^{-} (\sum_{i=1}^{m} v_{i} x_{ij}) = \sum_{i=1}^{m} v_{i} x_{ij}, \quad \forall j$$

Model problem (2) above cannot be solved using non-linear constraints. To solve the above model, a new definition of the goal programming (GP) provided.

Fraction $\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$, should be increased by increasing the numerator or decreasing the denominator. As a result MOFP can be changed as follows:

$$\begin{array}{ll} \text{Min} & \sum_{j=1}^{n} (\varphi_{j}^{-} + \varphi_{j}^{+}) & (3) \\ \text{S.t.} & \frac{\sum_{r=1}^{s} u_{r} y_{rj} + \varphi_{j}^{+}}{\sum_{i=1}^{m} v_{i} x_{ij} - \varphi_{j}^{-}} = 1 \quad , \quad \forall j & (3a) \end{array}$$

 $\frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 , \quad \forall j$ (3b) $\varphi_j^-, \varphi_j^+ \geq 0 , \quad v_i, u_r \geq \varepsilon , \quad \forall i, j, r$

Obviously, the constraints (3b) with respect to the constraints (3a) can model (3) is deleted.

Convert fractional linear programming model with constraints (4) is obtained:

$$\begin{array}{lll} \text{Min} & \sum_{j=1}^{n} (\varphi_j^- + \varphi_j^+) & (4) \\ \text{S.t.} & \sum_{j=1}^{s} u_r \, y_{rj} - \sum_{i=1}^{m} v_i \, X_{ij} + \varphi_j^- + \varphi_j^+ = 0, \ \forall \ j \\ \varphi_j^-, \varphi_j^+ \ge 0 & , \quad v_i \, , u_r \ge \varepsilon & , \ \forall i, j, r \end{array}$$

In simple linear programming model to replace the $\varphi_i^- + \varphi_i^+$ with φ_i following is obtained:

$$\begin{array}{ll} \text{Min } & \sum_{j=1}^{n} \varphi_{j} & (5) \\ \text{S.t. } & \sum_{j=1}^{s} u_{r} \, \mathbf{y}_{rj} \cdot \sum_{i=1}^{m} v_{i} \, \mathbf{X}_{ij} \, + \varphi_{j} = 0 &, \quad \forall \, j \\ & \varphi_{j} \geq 0 &, \quad v_{i} \, , \, u_{r} \geq \varepsilon &, \quad \forall i, j, r \end{array}$$

DMU_j, j = 1,...n Most respondents (Efficient), respectively. If and only if model (b) $\varphi_j = 0$, j= 1,...n So if we assume that $(u_r^*, v_i^*, \phi_j^*), \forall i, j, r$ the optimal

solution of the model (b) the level of efficiency of DMU_j , j = 1,...n can be obtained as follows:

$$\theta_{j}^{*} = \frac{\sum_{r=1}^{S} u_{r}^{*} y_{rj}}{\sum_{i=1}^{m} v_{i}^{*} x_{ij}} = 1 - \frac{\varphi_{j}^{*}}{\sum_{i=1}^{m} v_{i}^{*} x_{ij}} , \quad \forall j$$
 (6)

 DMU_j , j = 1,...n Most respondents (efficient) if and only if the model is (6) θ_i^* = 1 , j =1,...n

Suppose, central system consists of n, DMU have independent. Each DMU_j, j = 1,...n using the m input , X_{ij} \in R⁺, (i = 1, ..., m, j = 1, ..., n), s output Y_{ij} \in R⁺, (j = 1, ..., n, r = 1, ..., s) produce the system assumes that a centralized organization q , F_k \in R⁺, k = 1, ..., q holds excess supply and will allocate resources to each DMU. Accordingly, the expectations of the fixed output is P.

 $G_w \in R^+$, w = 1,...,p As goals are set for each DMU. Non-negative variables DMU_j are $\overline{f_{kj}}$ and $\overline{g_{wj}}$ assign input and output allocation

So this $\sum_{j=1}^{n} \overline{f_{k_j}} = F_k$, $\forall k \quad \sum_{j=1}^{n} \overline{g_{w_j}} = G_w$, $\forall w$ should be connected. We hence we get the following system:

$$\begin{array}{ll} \frac{\sum_{j=1}^{S} u_{r} y_{rj} + \sum_{w=1}^{D} u_{s+w} \overline{g_{wj}}}{\sum_{i=1}^{m} v_{i} X_{ij} + \sum_{k=1}^{q} v_{m+k} \overline{f_{kj}}} = 1 \quad , \quad \forall \; j \qquad (7a) \\ \sum_{j=1}^{n} \overline{f_{kj}} = F_{k} \quad , \quad \forall k \qquad (7b) \qquad (7) \\ \sum_{j=1}^{n} \overline{g_{wj}} = G_{w} \quad , \quad \forall w \qquad (7c) \\ u_{r} \; , \; u_{s+w} \; , \; v_{i_{-}} v_{m+k} \geq \epsilon \quad , \quad \overline{f_{kj}} \; , \; \overline{g_{wj}} \geq \\ 0 \quad , \quad \forall \; r, \; i, \; k, \; w, \; j \end{array}$$

This systems constraint (7a) guarantees that each DMU efficiency rating according to the input allocated surplus and additional output, would be one. Constraints (7b) and (7c) and the total resources allocated to the production targets, F_k and G_w are equal to the non-linearity of (7) we change variables in the following form.

$$u_{s+w} \overline{g_{wj}} = g_{wj}$$
$$v_{m+k} \overline{f_{kj}} = f_{kj}$$

The system (7), resulting the following system.

Multiples λ_j , μ_j to determine all inputs and all outputs excess surplus is defined. $\lambda_j F_k$, $\mu_j G_w$ We can use it for assigning the input and output setting of DMU, j.

When we use multiples x_j , μ_j It is possible that the system (8) is impossible. Additional variables to solve a linear programming model based on the planned target (GP) are defined. We define the positive and negative deviation variables for f_{kj} that g_{wj} is shown by $(\alpha_{kj}^-, \alpha_{kj}^+)$ and $(\beta_{wj}^-, \beta_{wj}^+)$.

$$\begin{array}{ll} \sum_{j=1}^{n} g_{wj} &= u_{s+w} G_{w} &, \quad \forall w \\ u_{r} , u_{s+w} , v_{i} , v_{m+k} \geq & \epsilon, \\ f_{ki} , g_{wi} , \alpha_{ki}^{-} , \alpha_{ki}^{+} , \beta_{wi}^{-} , \beta_{wi}^{+} \geq 0 , \forall r, i, k, w, j \end{array}$$

Equation (9) is an equation of fractional programming and it can be transformed into the following linear programming problem.

. 、

$$\begin{split} & \text{Min} \quad \sum_{i=1}^{n} \left(\sum_{k=1}^{q} (\alpha_{kj}^{-}, \alpha_{kj}^{+}) + \sum_{w=1}^{p} (\beta_{wj}^{-}, \beta_{wj}^{+}) \right) \\ & \text{S.t.} \quad \sum_{r=1}^{s} u_{r} y_{rj} + \sum_{w=1}^{p} g_{wj} \cdot \left(\sum_{i=1}^{m} v_{i} X_{ij} + \sum_{k=1}^{q} f_{kj} \right) \\ & = 0 \\ & f_{kj} + \alpha_{kj}^{-} - \alpha_{kj}^{+} = v_{m+k} \lambda_{j} F_{k} , \quad \forall k, j \\ & (10) \\ & g_{wj} + \beta_{wj}^{-} - \beta_{wj}^{+} = u_{s+w} \mu_{j} G_{w} , \quad \forall w, j \\ & \sum_{j=1}^{n} f_{kj} = v_{m+k} F_{k} , \quad \forall k \\ & \sum_{j=1}^{n} g_{wj} = u_{s+w} G_{w} , \quad \forall w \\ & u_{r} , u_{s+w} , v_{i} , v_{m+k} \geq \epsilon , \\ & f_{kj} , g_{wj} , \alpha_{kj}^{-}, \alpha_{kj}^{+}, \beta_{wj}^{-}, \beta_{wj}^{+} \geq 0 , \forall r, i, k, w, j \end{split}$$

So the model (1) enables us to DMUs efficiency of resource allocation and goal setting check out. The presence of these additional inputs and outputs of the model (1) can be transformed into the following model.

Max

$$\begin{array}{l} F^{=} \\ \begin{cases} \frac{\sum_{r=1}^{s} u_{r} y_{r1} + \sum_{w=1}^{p} u_{s+w} \overline{g_{w1}}}{\sum_{i=1}^{m} v_{i} x_{i1} + \sum_{k=1}^{q} v_{m+k} \overline{f_{k1}}}, \frac{\sum_{r=1}^{s} u_{r} y_{r2} + \sum_{w=1}^{p} u_{s+w} \overline{g_{w2}}}{\sum_{i=1}^{m} v_{i} x_{i2} + \sum_{k=1}^{q} v_{m+k} \overline{f_{k2}}}, \ldots \\ & , \frac{\sum_{r=1}^{s} u_{r} y_{m} + \sum_{w=1}^{p} u_{s+w} \overline{g_{wn}}}{\sum_{i=1}^{m} v_{i} x_{i1} + \sum_{k=1}^{q} v_{m+k} \overline{f_{kn}}} \end{array} \right) \\ \text{S.t.} \quad \begin{array}{c} \frac{\sum_{r=1}^{s} u_{r} y_{rj} + \sum_{w=1}^{p} g_{wj}}{\sum_{i=1}^{m} v_{i} x_{ij} + \sum_{k=1}^{q} f_{kj}} \leq 1 \ , \ \forall j \qquad (11) \\ & u_{r} \ , \ u_{s+w} \ , \ v_{i} \ , v_{m+k} \geq \epsilon \ , \qquad \overline{f_{kj}} \ , \overline{g_{wj}} \geq \\ 0 \ , \ \forall r, i, k, w, j \end{array}$$

4. Ranking based on common weights

4.1. Numerical examples

In this section, two numerical examples are presented for the models used. In the first example, the hypothesis proposed by the cook and kress (1999) to examine resource allocation.

In this example, the number 12, we have DMU. Three-input $\{X_1, X_2, X_3\}$ and two output $\{Y_1, Y_2\}$ is presented in Table 1. As shown in Table 1, Unit 9 has an efficient at one.

In Table 2, DMU_9 the highest amount allocated 16.330 gets compared to others, because the functionality is DMU_9 before allocating costs. Similarly, because DMU_7 the DMU_{11} worst

performance of the series is produced in this way have received a minimum of zero is assigned to them. In Table 3, the ranking of units is discussed.

obtain the best results and other work units may also be given points according to Table 3, ranked.

Given the issues raised in the concluding DMU_o 12, DMU raised its best performance, In order to

Table 1: inputs and output data							
DMU	X ₁	X2	X ₃	Y ₁	Y ₂	Efficiency b	efore allocation
						Cook, Kress (1999)	Suggested method
1	350	39	9	67	751	0.757	0.649
2	298	26	8	73	611	0.926	0.641
3	422	31	7	75	584	0.746	0.439
4	281	16	9	70	665	1.000	0.736
5	301	16	6	75	445	1.000	0.488
6	360	29	17	83	1070	0.961	0.892
7	540	18	10	72	457	0.862	0.279
8	276	33	5	78	590	1.000	0.672
9	323	25	5	75	1074	1.000	1.000
10	444	64	6	74	1072	0.833	0.713
11	323	25	5	25	350	0.333	0.326
12	444	64	6	104	1199	1.000	0.810

Table 2: results of resource allocation using cook and Kress (1999) method

DMU	Cook, Kress	Beasley	Cook, Zhu	Suggested method
1	14.520	6.780	11.220	8.199
2	6.740	7.210	0.000	7.462
3	9.320	6.830	16.950	4.284
4	5.600	8.470	0.000	9.301
5	5.790	7.080	0.000	4.807
6	8.150	10.060	15.430	15.370
7	8.860	5.090	0.000	0.000
8	6.260	7.740	0.000	7.339
9	7.310	15.110	17.620	16.330
10	10.080	10.080	21.150	11.598
11	7.310	1.580	17.620	0.000
12	10.080	13.970	0.000	15.310
Sum	100.020	100.000	99.990	100.000

Table3: Ranking						
DMU	Suggested method	Unit ranking				
1	8.199	7				
2	7.462	8				
3	4.284	10				
4	9.301	4				
5	4.807	9				
6	15.370	2				
7	0.000	12				
8	7.339	6				
9	16.330	1				
10	11.598	5				
11	0.000	11				
12	15.310	3				
Sum	100.000					

In the second instance, in this example 20, we have DMU. Three inputs and three outputs are presented in Table 4 (Amirteimoori and Mohaghegh Tabar, 2010).

We assume that a manager has decided to allocate 175 DMUs unit, so that the target unit 620 receives the output. For example (F_1 =175, G_1 = 620).

Columns 2 and 4 of Table 4 show the ratings before and after of the efficient allocation of resources. Similarly, columns 6 and 8, some of the

resources of the target output DMUs assign. According to our model, with using model 5 and model 6 first we calculate the efficiency of DMUs, which is presented in column 3 of Table 4.

The model 10 shows the optimization of resource allocation and the selection of targets for each unit and results in columns 7 and 9 in the table provided.

Table 4 shows that $\overline{f_{1j}}$ and $\overline{g_{1j}}$ · j=1,...,20 represents the assignment of the new input and output of each DMU.

If we evaluate DMUs through our model 5 and 6, With additional input and output $\overline{f_{11}}$ and $\overline{g_{11}}$ all DMUs are efficient.

Table4: Resulting efficiency, resource allocation and target setting									
DMU _j	Efficiency before allocation		Efficiency after allocation		Resource allocation (f ₁₁)		Target setting (g ₁₁)		
	AM method	Proposed method	AM method	Proposed method	AM method	Proposed method	AM method	Proposed method	
1	1.000	0.796	1.000	1.000	13	6.118	28	28.970	
2	0.711	0.675	0.711	1.000	4	0.000	20	23.076	
3	0.896	0.485	0.896	1.000	11	0.000	9	41.779	
4	0.596	0.501	0.598	1.000	7	0.000	9	29.145	
5	1.000	0.478	1.000	1.000	11	0.000	6	28.825	
6	1.000	1.000	1.000	1.000	0	4.364	21	0.000	
7	0.704	0.597	0.704	1.000	11	0.000	6	22.133	
8	1.000	0.940	1.000	1.000	0	3.202	14	13.878	
9	1.000	1.000	1.000	1.000	0	4.063	22	14.984	
10	0.523	0.282	0.530	1.000	10	0.000	5	21.509	
11	0.668	0.480	0.776	1.000	9	0.000	27	52.552	
12	1.000	0.748	1.000	1.000	0	0.000	25	20.885	
13	0.958	0.910	1.000	1.000	0	6.099	22	11.018	
14	0.994	0.562	1.000	1.000	0	0.000	27	32.011	
15	1.000	1.000	1.000	1.000	6	12.591	61	51.634	
16	1.000	1.000	1.000	1.000	18	119.404	82	0.000	
17	0.942	0.793	0.951	1.000	34	0.000	72	67.222	
18	1.000	0.757	1.000	1.000	6	0.000	56	65.325	
19	1.000	0.926	1.000	1.000	13	19.159	68	62.655	
20	0.891	0.809	0.891	1.000	21	0.000	39	32.409	
				Sum	174	175	619	620	

As a result of the model presented, this example indicates that, to be able to achieve our goal, we combine Resource allocation and selection of target output.

5. Conclusion

In this paper, has been studied the relationship between resource allocation problems with the problem of weight control and target selection.

Calculation of common weights causes that hardly we have been than one efficient unit. In this case it is possible to rank the efficient DMUs using the efficiency calculated by common weights.

We propose an alternative mathematical model to allocate the fixed resources to the units along with setting the expected common increase of the targets to the units in a fair way.

Also the optimal solution of the proposed model always assigns an efficiency score of unity to all DMUs.

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